

## Sample Interest Rate Problems:

Introductory Note: Any interest rate problem involves several or all of the following variables:

$N$  = the number of periods in which benefits or costs are being received or paid

$I/YR$  = the interest rate per period applicable to the value of benefits and costs

$PV$  = the present value of future benefits or in some cases costs at  $I/YR$

$PMT$  = the periodic payment of funds over time [if the payment is constant]

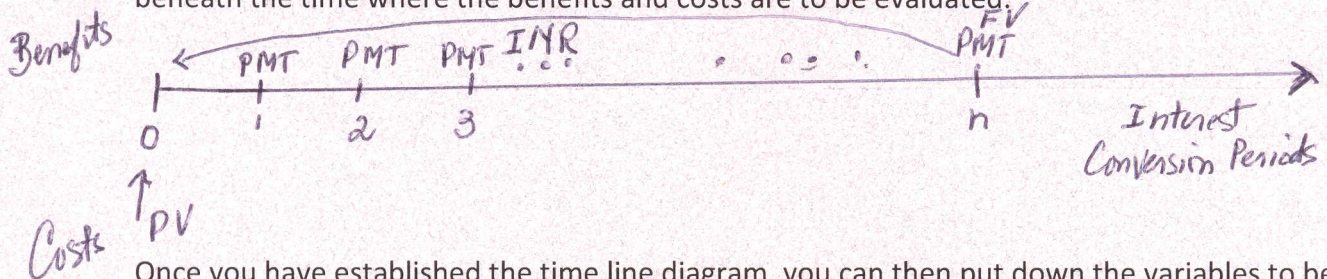
$CF$  = cashflow representing the periodic payment of either an even or uneven amount

$FV$  = the future value of benefits or in some cases costs at  $I/YR$

Interest rate problems can be boiled down to being given three pieces of information and solving for the fourth or having four pieces of information and finding the fifth. Consequently, one good starting place in solving an interest problem is to write down which variables you are given, and the one you are trying to solve for.

The most significant factor in successfully solving an interest rate problem is creating an accurate time line diagram that includes the variables that have been given, the one to be solved for, and the number of periods and appropriate rate of interest based on interest conversion periods. An interest conversion period is the length of time to earn interest. For example, if the problem says you are using a 10% interest rate compounded semi-annually for 10 years, then the number of interest conversion periods is: 10 years  $\times$  2 interest conversions per year = 20 periods, and the appropriate interest rate is  $10\%[\text{annual rate}]/2 = 5\%$  per interest conversion period [i.e. each half year].

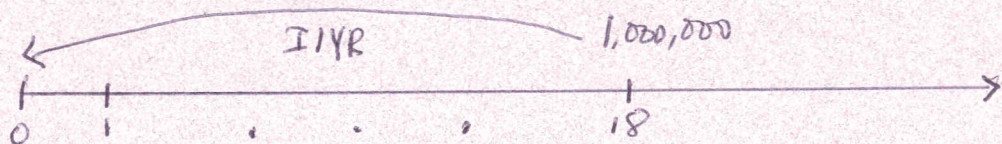
In constructing a time line diagram, benefits go on top [and are generally given positive value], costs go below [and are generally given negative value –i.e. you are expending money to buy the project or financial investment], the interest rate goes on top, and there is an arrow beneath the time where the benefits and costs are to be evaluated.



Once you have established the time line diagram, you can then put down the variables to be placed into the financial calculator and solve for the one variable you're looking for.

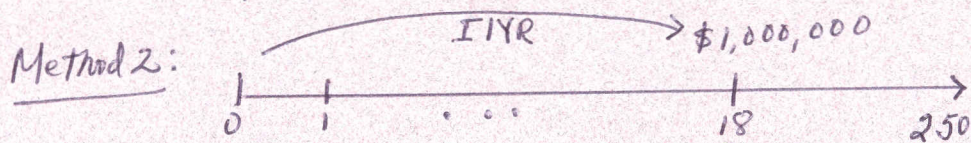
When solving these types of problems, it is important to clear all the registers in your calculator prior to inputting the variables. For example, with the HP 10BII you will want to push the gold, C [i.e., Clear All] key to zero out the financial registers. This initial step will prevent information from a prior problem remaining in a financial register and causing you to get an incorrect answer to your problem.

2-3 Your parents will retire in 18 years. They currently have \$250,000 and they think they will need \$1,000,000 at retirement. What annual interest rate must they earn to reach their goal, assuming they don't save any additional funds.



- 250,000

Method 1:  $N=18$ ;  $PV = -250,000$ ,  $FV = 1,000,000$   
 $IYR = 8.0059 \approx 8\%$



$N=18$ ;  $PV = -250,000$   
 $FV = 1,000,000$ ,  $IYR = 8\%$

$$250,000 (1+i)^{18} = 1,000,000$$

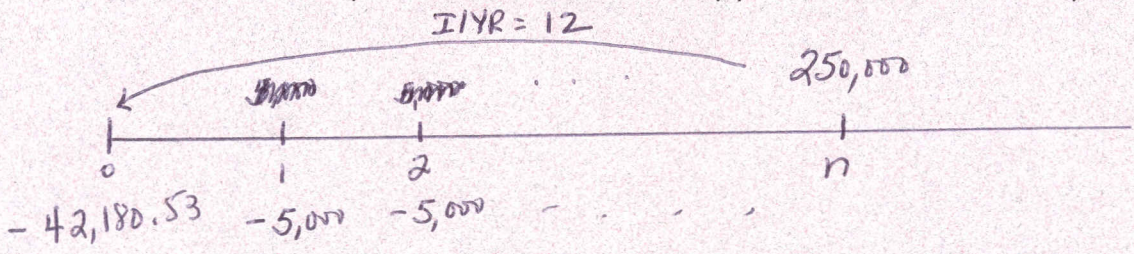
$$(1+i)^{18} = 4$$

$$(1+i) = (4)^{1/18}$$

$$(1+i) = 1.08$$

$$i = .08 \text{ or } 8\%$$

2-4 You have \$42,180.53 in a brokerage account, and you plan to deposit an additional \$5,000 at the end of every future year until your account totals \$250,000. You expect to earn 12% annually on the account. How many years will it take to reach your goal?



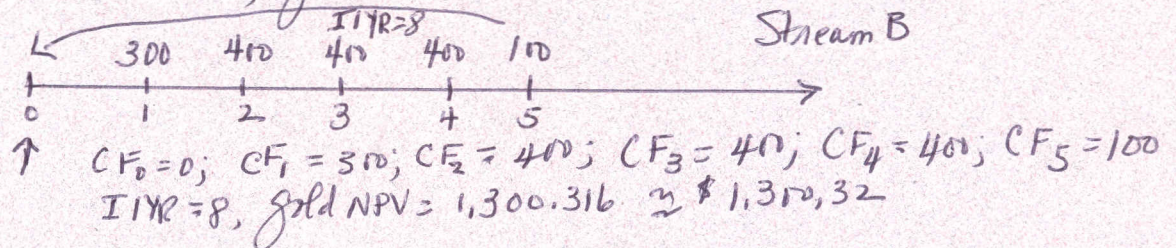
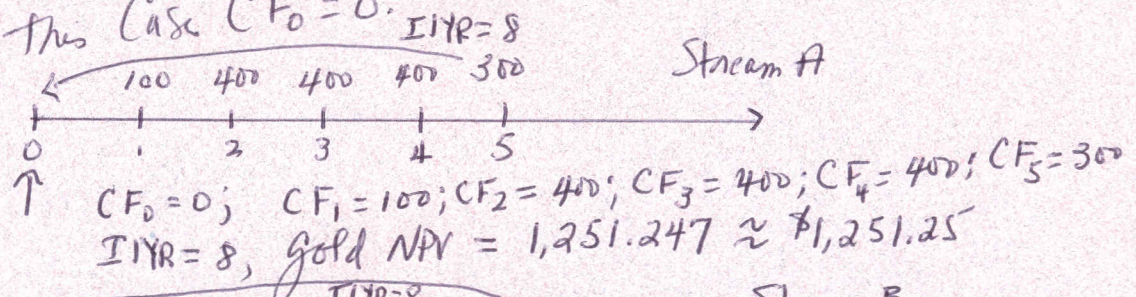
↑

$IYR = 12$   
 $PV = -42,180.53$   
 $PMT = -5,000$   
 $FV = 250,000$   
 $N = 11$

- 2-14 a. Find the present values of the following cash flow streams. The appropriate interest rate is 8%. (Hint: It is fairly easy to work this problem dealing with the individual cash flows. However, if you have a financial calculator, read the section of the manual that describes how to enter cash flows such as the ones in this problem. This will take a little time, but the investment will pay huge dividends throughout this course. Note, if you do work with the cash flow register, then you must enter a value for the initial cash flow  $CF_0$ , in this case  $CF_0 = 0$ ).

year	Cash Stream A	Cash Stream B
1	100	300
2	400	400
3	400	400
4	400	400
5	300	100

The cash flows represent uneven amounts (not a constant payment) so you need to use the Cash flow key. (3<sup>rd</sup> row from top, 3<sup>rd</sup> key from left on HP12B) for these calculations. The financial calculator is set up to take a number for the initial cash flow,  $CF_0$  (which for some problems is a negative number  $\Rightarrow$  representing a cost) in this case  $CF_0 = 0$ .



- b. What is the value of each cash flow at a 0% interest rate?

In both cases the p.v. of the cashflows is equal to the sum of the benefits.

Stream A  $\frac{100}{(1+0)} + \frac{400}{(1+0)^2} + \frac{400}{(1+0)^3} + \frac{400}{(1+0)^4} + \frac{300}{(1+0)^5} = \frac{100+400+400+400+300}{1} = \$1,600$

Stream B  $\frac{300}{(1+0)} + \frac{400}{(1+0)^2} + \frac{400}{(1+0)^3} + \frac{400}{(1+0)^4} + \frac{100}{(1+0)^5} = \$1,600$

2-19 Universal Bank pays 7% interest, compounded annually, on time deposits. Regional Bank pays 6% interest, compounded quarterly.

a. Based on effective interest rates, in which bank would you prefer to deposit your money?

To answer this question Consider an initial deposit of \$100, to be invested 1 year.

Universal Bank

Timeline: 0 to 1 year. IYR = 7%. PV = -100, FV = 107.

Regional Bank: Convert time line to Quarters; 4 quarters. Quarterly Rate = 6/4.

Timeline: 0 to 4 quarters. IYR = 6/4. PV = -100, FV = 106.136.

Effective Rate of Interest =  $\frac{107 - 100}{100} = 0.07$

Effective Rate of Interest = 6.136%

∴ you would prefer Universal's 7% Annual Rate

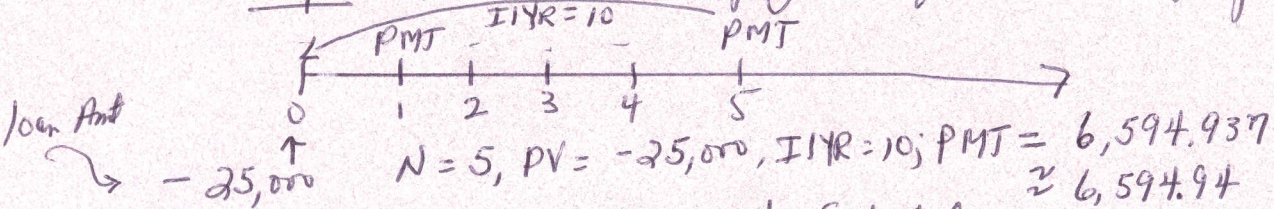
b. Could your choice of banks be influenced by the fact that you might want to withdraw your funds during the year, as opposed to, the end of the year? In answering this question, assume that funds must be left on deposit during the entire compounding period in order for you to receive any interest.

If funds must be left on deposit until the end of the compounding period (1 year for Universal and 1 quarter for Regional, and you think there is a high probability that you will make a withdrawal during the year, the Regional account might be preferable.

2. If you withdraw 364 days after depositing money in Universal you would receive no interest, however if you were to have deposited with Regional you would have received 3 quarterly interest payments.

Set up an amortization schedule for a \$25,000 loan to be repaid in equal installments at the end of each of the next 5 years. The interest rate is 10%.

Step 1: Find the annual payment from the Bank's perspective

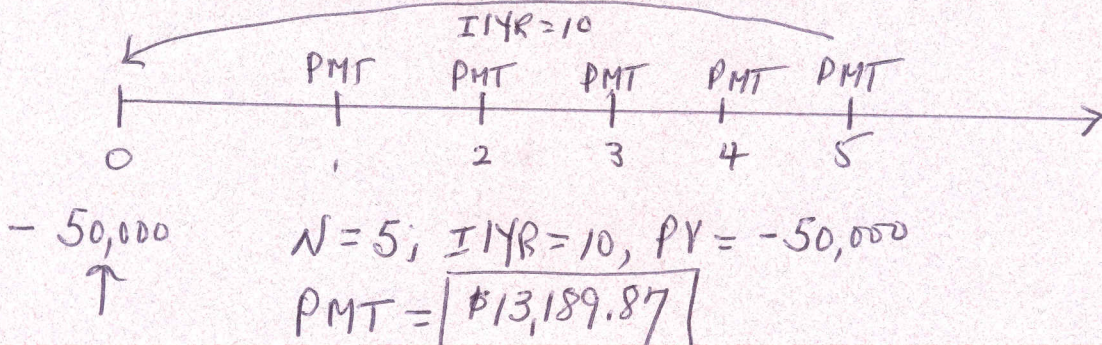


Step 2 Develop The Amortization Schedule

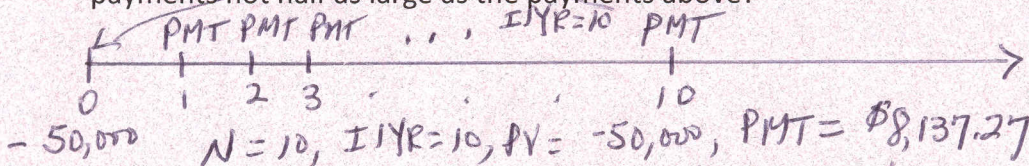
Period	Payment	Interest	Principal Repaid	Outstanding loan Balance
0				\$25,000
1	6,594.94	$25,000 \times 10$ $= 2,500$	$6,594.94 - 2,500$ $= 4,094.94$	$25,000 - 4,094.94$ $= 20,905.06$
2	6,594.94	$20,905.06 \times 10$ $= 2,090.51$	$6,594.94 - 2,090.51$ $= 4,504.43$	$20,905.06 - 4,504.43$ $= 16,400.63$
3	6,594.94	$16,400.63 \times 10$ $= 1,640.06$	4,954.88	11,445.75
4	6,594.94	1,144.58	5,450.36	5,995.39
5	6,594.94	599.54	5,995.39	0
	<u>\$32,974.69</u>	<u>7,974.69</u>	<u>25,000.00</u>	

last payment must be \$6,594.93 due to rounding error

How large must each annual payment be if the loan is for \$50,000? Assume the interest rate remains at 10% and the loan is paid off over 5 years.

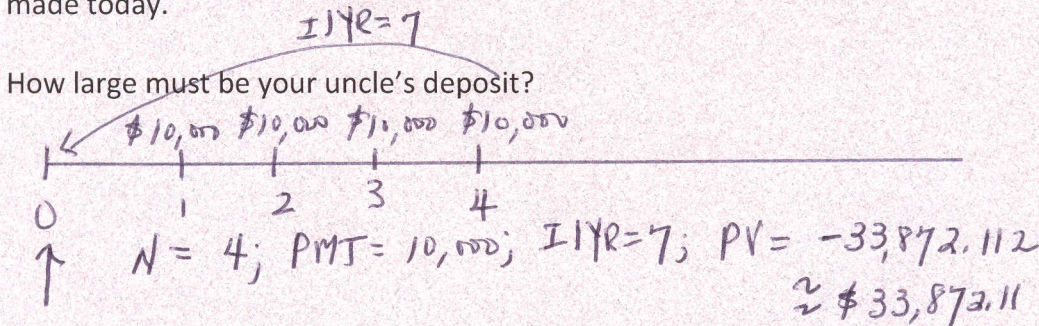


How large must each payment be if the loan is for \$50,000, the interest rate is 10%, and the loan is paid off in equal installments at the end of each of the next 10 years? This loan is for the same amount as the loan above, but the payments are spread out over twice as many periods. Why are these payments not half as large as the payments above?



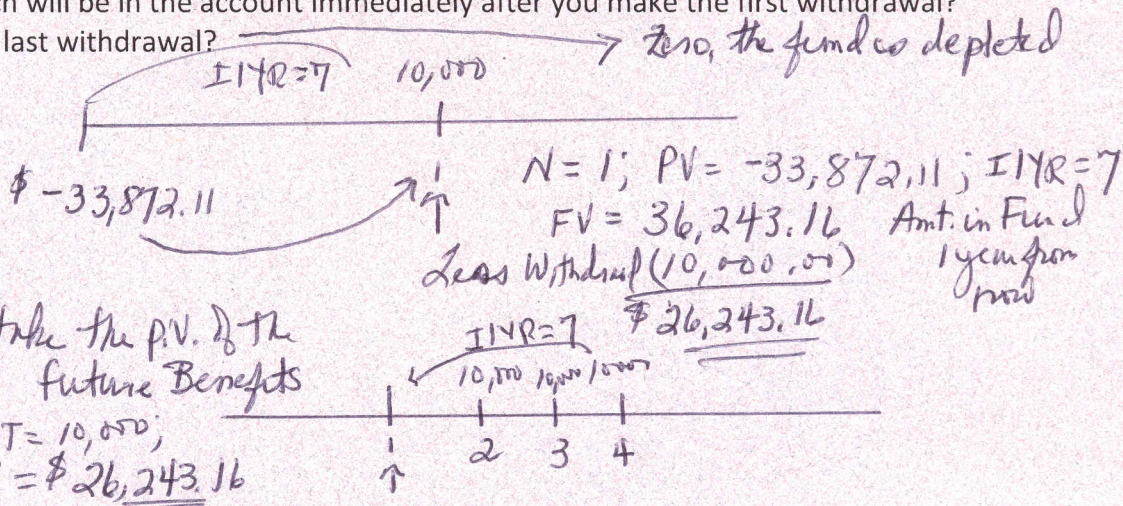
↑ Because the payments are spread out over a longer period of time, more interest will be paid on the loan and thus a higher payment required due to compounding of interest.

2-24 To complete your last year in business school and then go through law school [onward and upward] you will need \$10,000 per year for 4 years, starting next year [that is, you will need to withdraw the first \$10,000 one year from today]. Your rich uncle offers to put you through school, and he will deposit in a bank paying 7% interest a sum of money that is sufficient to provide the 4 payments of \$10,000 each. His deposit will be made today.



Amt your Supportive Uncle needs to provide

How much will be in the account immediately after you make the first withdrawal? After the last withdrawal?

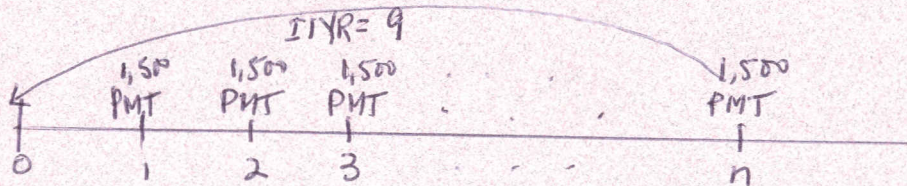


Retrospective Loan Balance → Method 1

Prospective Loan Balance → Method 2

take the P.V. of the future Benefits

2-25 While Mary Corens was a student at the University of Tennessee, she borrowed \$12,000 in student loans at an annual interest rate of 9% [Yikes!]. If Mary repays \$1,500 per year, how long, to the nearest year, will it take her to repay this loan?



-12,000

↑

Amt. of Student Loan

$$I/YR = 9$$

$$PV = -12,000$$

$$PMT = 1,500$$

$$N = \boxed{14.77 \text{ years}}$$

≈ 15 years

Unless she has a rich uncle